Phillips Curves in Noisy Information Forecasts

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Abstract

Does the average professional forecaster in the U.S. have a Phillips curve in mind when reporting survey forecasts? Does that perception change over time? First, I show that the average forecaster has a Phillips curve in mind and that her perceived slope coefficient is time-varying, having become flatter over the past decades. Second, uncertainty about the present state of inflation and unemployment might distort inference about forecaster's perception concerning the Phillips curve slope. I show that failing to control for the presence of these information frictions in survey forecasts of inflation and unemployment can significantly underestimate the perceived slope. Last, I analyze the state-dependency of the average forecaster's perceived Phillips curve slope. Here, I find that it is negatively correlated with the forecaster's attention to new information on inflation, consistent with predictions of imperfect information models.

Keywords: Expectation Formation, Non-Linear State-Space Models, Phillips Curve

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1 Introduction

Macroeconomic models are founded on the belief that agents in the economy act and form expectations based on well established economic principles. Consumers consume more today when the expected real rate decreases. Price-setters set higher prices when there is more demand for the respective good. Such behavior should result in observed macroeconomic outcomes consistent with individual actions and expectations. However, do agents in the economy internalize well-known macroeconomic principles? In that spirit, one concept which has been at the core of macroeconomics for some decades, the Phillips curve, deserves special attention. Not only did its micro-foundations change considerably over time but these changes were often linked with advances in the way economists thought about the properties of expectation formation.[1] This paper attempts to understand how agents perceive the response of inflation to changes in unemployment, and how that perception has been evolving over the past decades. More specifically, this paper focuses on the average professional forecaster in the U.S. and asks if her perceived law of motion or forecasting model of inflation and unemployment is consistent with the Phillips curve. Concerning the importance of the properties of expectation formation for the Phillips curve, Coibion et al. (2018) call for a careful reconsideration of the expectation formation process and the use of survey data in macroeconomic analysis. In that respect, this paper also sheds some light on the interactions of the perceived Phillips curve and the structure of the average forecaster's information set. It does this by analyzing the co-movement in the perceived Phillips curve slope and the frequency of inflation and unemployment forecast updating.

However, determining if the average professional forecaster has a Phillips curve in mind raises some challenges in the presence of information frictions in surveys of forecasts on inflation and unemployment. When forecasters receive noisy signals about the present state of the economy, their perceived uncertainty about inflation and unemployment might lead to sluggish updating of their survey forecasts concerning these variables. Therefore, even if forecasters think that a Phillips curve relation exists in the data, and this is reflected in their forecasting model, their survey responses might not reveal that. To be more specific, suppose that forecasters are more uncertain about the present state of inflation than

[1] See Mankiw and Reis (2010) for an overview.

that of unemployment. In response to a surprise increase in unemployment, forecasters might, therefore, update their unemployment forecasts as expected. Nevertheless, since they perceive signals pointing to falling inflation as "too" noisy, they might not update their inflation forecasts.[2] In such a scenario, an econometrician trying to detect a Phillips curve relation in survey data by looking at the correlation of inflation and unemployment survey forecasts, would not be able to find any significant correlation. Indeed, Coibion and Gorodnichenko (2015) document that survey forecasts do exhibit large degrees of information frictions and that these differ across variables. Therefore, an issue as mentioned above, might arise.

To estimate the perceived Phillips curve slope of the average professional forecaster, I set up and estimate an empirical model of the average forecaster's survey responses of inflation and unemployment, in which an explicit role is given to the degrees of information frictions in both variables. To that purpose, I follow Mertens and Nason (2017) who set up a joint model of inflation real-time data and inflation survey expectations, by combining the unobserved component model of inflation with stochastic volatility by Stock and Watson (2007) with a sticky information law of motion for expectation formation as in Coibion and Gorodnichenko (2015). I extend their framework in two ways. First, as the average professional forecaster's forecasting model, I make use of a bi-variate trend-cycle decomposition of inflation and unemployment by Chan et al. (2016). Second, I allow for a richer structure of information frictions with differing frequencies of forecast updating for inflation and unemployment, respectively. As there is ample evidence on the time-varying nature of the Phillips curve slope and the degrees of information frictions[3], I also allow for time-variation in parameters as in Mertens and Nason (2017). The interaction of the term structure of inflation and unemployment survey forecasts together with the explicit estimation of the degrees of information friction both for inflation and unemployment makes it possible to uncover the average forecaster's perceived Phillips curve slope.

I find that the average professional forecaster's survey responses of inflation and un-

[2] In the empirical model used in this paper surprise innovations to unemployment affect inflation contemporaneously, but not the other way around. This can be perceived as an identifying assumption for aggregate demand shocks and their impact on inflation, as therefore measured by the Phillips curve slope. These identifying assumptions are up for debate and are not the focus of this paper.

[3] For example Kuttner and Robinson (2010) for the Phillips curve slope, and Coibion and Gorodnichenko (2015) and Mertens and Nason (2017) for information frictions in inflation.

employment are consistent with a forecasting model incorporating a Phillips curve slope. Moreover, the slope coefficient varies over time. More precisely, the Phillips curve was flat in the 1970s, became stepper towards 1985, and slowly became flat again during the 1990s and 2000s. Furthermore, I show that failing to account for the varying degrees of information frictions in inflation and unemployment can underestimate the perceived slope coefficient by up to 60% in absolute values.

Also, I analyze the state-dependency of the perceived Phillips curve slope with respect to parameters and states of the empirical model. I find that the slope coefficient is negatively correlated with trend inflation, supporting sticky-price models. That is similar to findings in Ball and Mazumder (2011), which show that the negative correlation of trend inflation and the slope coefficient holds true for a Phillips curve based on observed inflation and unemployment outcomes. However, holding inflation trend and inflation volatility constant and adding a measure of forecaster's attention to inflation data, I additionally find that forecaster's attention negatively co-varies with the perceived Phillips curve slope of professional forecasters, while trend inflation becomes irrelevant as an explanatory factor for the perceived Phillips curve slope. This implies that as forecasters become more precise in their knowledge about the present state of inflation, their perceived Phillips curve becomes steeper.

I contribute to the literature along multiple dimensions. First, this paper extends the analysis of inflation survey expectations to a multivariate setting, by explicitly taking into account unemployment survey forecasts' dynamics and their relation to inflation survey forecasts. In a model comparison exercise Coibion and Gorodnichenko (2015) argue that an AR(1) process for inflation is sufficient to characterize predictability of forecasts errors as compared to a VAR including unemployment. My results differ in that respect. I find that an empirical model describing data and survey data on inflation and unemployment exhibits a superior model fit over an empirical model treating both variables individually. This might be either due to this paper's use of a trend-cycle decomposition or because of the relevance of time-variation in the Phillips curve slope. Andrade et al. (2016) also make use of a multivariate model of survey forecasts with information frictions, but they focus on the dynamics of disagreement among professional forecasts. Similar in the spirit of uncovering perceptions about states and parameters in forecasters' models by use of multi-level forecasts and state-space systems is Jain (2017), who provides estimates of forecasters' perceived inflation persistence and Krane (2011) who identifies forecasters' perceptions of permanent and transitory shocks to GDP. Further, Patton and Timmermann (2011) provide an unobserved component model for both inflation and output growth survey forecasts and use their model to extract information on the predictability of inflation and output growth and the importance of measurement errors.

Second, various papers in the literature establish that survey measures of inflation, unemployment, and wage rates are consistent or in line with a Phillips curve. See for example Tillmann (2010), Fendel et al. (2011), and Dräger et al. (2016), and others.[4] Using the empirical model introduced in this paper, I deviate from these papers by explicitly giving a role to information frictions in survey forecasts, quantifying their magnitude, and showing how these frictions might distort evidence on the model- or theory-consistency of survey forecasts.

Finally, this paper provides empirical support of a general property of Phillips curve specifications based on deviations from full information rational expectation. In these models, the strength of agents' reaction to changes in aggregate demand usually depends on parameters governing the degree of information frictions. For instance, in models of rational inattention such as those in Maćkowiak and Wiederholt (2009, 2015), the degree of information friction is endogenous and is directly linked to agent's perceived signal-to-noise ratio, which influences price-setters' reaction to aggregate conditions. Similarly, in models of sticky information such as in Mankiw and Reis (2002), the sticky information Phillips curve slope is, among other parameters, governed by the exogenous information arrival frequency of agents' information sets. The more frequently they receive new information about the state of the economy, the stronger agents react to movements in aggregate demand.

The rest of this paper is structured as follows: Section 2 provides intuition regarding how the presence of information frictions can distort inference about forecasters' perception concerning their Phillips curve slope. Section 3 introduces the perceived model

[4] Pierdzioch et al. (2011) and Ball et al. (2014) provide evidence that survey forecasts of output and unemployment are consistent with Okun's law. Mitchell and Pearce (2010) ask if Wall Street economists believe in Okun's law and the Taylor Rule. Carvalho and Nechio (2014) show that households in the Michigan Survey indeed understand basic features of monetary policy, meaning they report expectations consistent with a Taylor-type monetary policy rule.

for professional forecasters and their law of motion of expectation formation. Section 4 discusses the data and empirical methodology. Section 5 presents results, and Section 6 concludes.

2 The Perceived Phillips Curve Slope and Information Frictions

The purpose of this section is twofold. First, using a small econometric toy model, I aim to show which role information frictions play, when attempting to uncover the 'belief" a forecaster has about the Phillips curve slope. Second, this section lays out, which components will be necessary within an empirical model to identify the perceived slope coefficient.

For a direct approach to asses if survey forecasts are compatible with a Phillips curve model, one might estimate an equation of the following type

$$F_t \pi_{t+h} = g\left(F_t u_{t+h}\right) + e_t, \tag{1}$$

where $F_t \pi_{t+h}$ and $F_t u_{t+h}$ represent survey responses of varying forecast horizon h, and g is a function of the right-hand side regressors. Let us assume for simplicity that $g\left(F_t u_{t+h}\right) = \gamma^{SPF} F_t u_{t+h}$,[5] and let us focus on now-cast responses, i.e. $h = 0$, which leads to the specification

$$F_t \pi_t = \gamma^{SPF} F_t u_t + e_t. \tag{2}$$

Such regression specifications represent a common approach in the literature to asses model- or theory-consistency of survey forecasts.[6] Here the researcher is essentially regressing forecasts of one variable on forecasts of another variable, in the hope that such a regression can tell us something about both variables' relation in the mind of the forecaster. More specifically a negative statistically significant parameter γ^{SPF} would show that the survey data is at least consistent with some concept of the inflation-unemployment trade-off.

[5]SPF stands for Survey of Professional Forecasters, and should indicate that the parameter is estimated using only survey data.

[6]Compare for example Dräger et al. (2016) or Carvalho and Nechio (2014) for the Taylor rule.

The main point of this section is to show that given data on survey responses of inflation and unemployment, a 'direct regression' as given by equation (2), does not necessarily provide us with professional forecasters' belief about the Phillips curve slope as her forecasting model, or perceived model of the economy,[7] would imply. More precisely, let us assume that the forecasters' perceived law of motion is of the following specification

$$\pi_t = \gamma^{PLM} u_t + \epsilon_t^{\pi} \tag{3}$$

$$u_t = \epsilon_t^{u}, \tag{4}$$

which implies that unemployment u_t is driven by an i.i.d. process ϵ_t^u, and that inflation π_t is determined by contemporaneous unemployment via the Phillips curve slope γ^{PLM} and an i.i.d. innovation ϵ_t^{π}.

If forecasters receive noisy signals about inflation and unemployment, i.e. $s_{i,t}^{\pi}$ and $s_{i,t}^{u}$, average forecast responses result from the application of the Kalman filter to the underlying signal extraction problem, which leads to the following (simplified) representations of the Kalman mean updating equations for forecaster i

$$F_t \pi_t^{(i)} = (1 - \lambda^{\pi}) \underbrace{\left(\pi_t + \psi_{i,t}^{\pi}\right)}_{:= s_{i,t}^{\pi}} \tag{5}$$

$$F_t u_t^{(i)} = (1 - \lambda^{u}) \underbrace{\left(u_t + \psi_{i,t}^{u}\right)}_{:= s_{i,t}^{u}}, , \tag{6}$$

where $\lambda^{\pi}, \lambda^{u} \in (0, 1)$ are parameters governing the degrees of information frictions in inflation and unemployment, respectively. $F_t \pi_t^{(i)}$ and $F_t u_t^{(i)}$ are the survey now-casts of forecaster i for inflation and unemployment. $\psi_{i,t}^{\pi}$ and $\psi_{i,t}^{u}$ are noise realizations of inflation and unemployment. Equations (5) and (6) imply that individual survey forecasts will not only reflect information about the states of the economy, but also about the uncertainty concerning these states. As this paper deals with the average professional forecaster,

[7] 'Forecaster's forecasting model' and 'perceived law of motion' will be used interchangeably in this paper.

aggregating over all i, and assuming that noise realizations cancel out[8], we arrive at

$$F_t\pi_t = (1-\lambda^\pi)\,\pi_t \tag{7}$$

$$F_t u_t = (1-\lambda^u)\,u_t. \tag{8}$$

These representations of the law of motion of survey expectations are similar to Coibion and Gorodnichenko (2015), when assuming that forecasters have noisy information. As a result of noisy information, forecasters' survey responses do not fully reflect the impact of shocks on macroeconomic aggregates. Intuitively, the nosier signals about inflation and unemployment are, the less frequently survey forecasts will be updated.

Given these forecast smoothing equations, the average forecaster's perceived law of motion now implies that variation in inflation can be attributed to variation in unemployment and the innovation ϵ_t^π

$$F_t\pi_t = (1-\lambda^\pi)\left(\gamma^{PLM}u_t + \epsilon_t^\pi\right). \tag{9}$$

Plugging in the relation of unemployment data and unemployment now-cast (8) therefore yields

$$F_t\pi_t = \frac{1-\lambda^\pi}{1-\lambda^u}\gamma^{PLM}F_t u_t + e_t. \tag{10}$$

Comparing this to (2) gives us a direct mapping from the Phillips curve slope γ^{SPF}, we can infer from direct regressions using survey responses, to forecasters' perceived slope coefficient γ^{PLM} using

$$\gamma^{SPF} := Corr\left(F_t\pi_t, F_t u_t\right) = \frac{1-\lambda^\pi}{1-\lambda^u}\gamma^{PLM}. \tag{11}$$

This mapping is intuitively appealing. Suppose that in response to a one p.p. change in unemployment, inflation is reduced by -0.5 p.p. Using the forecasters' perceived law of motion, this would imply that $\gamma^{PLM} = -0.5$. Further suppose that forecasters update their unemployment survey responses fully to shocks, $\lambda^u = 0$, but adjust their inflation

[8]This follows from the law of large numbers if noise follows a normally distributed random variable with mean 0. This is a common assumption in the literature.

forecasts only partially, $\lambda^\pi = 0.9$. In such an environment, the unemployment survey response would be -1 p.p., whereas the inflation survey response amounts to -0.05 p.p., which implies that $\gamma^{SPF} = -0.05$. An econometrician observing survey responses would conclude that survey responses show quantitatively small reactions of inflation to changes in unemployment survey responses. However, this assessment should not be treated as an evaluation of the average forecaster's perceived slope coefficient. This example indicates that in a scenario where information frictions in inflation play a more significant role than similar frictions in unemployment, the estimates of the Phillips curve slope using survey responses potentially underestimates the perceived slope of forecasters in absolute values. Generally, as long as the degrees of information frictions differ across variables, i.e. $\lambda^\pi \neq \lambda^u$, direct regressions of inflation forecasts on unemployment forecasts alone do not identify the slope coefficient forecasters incorporate in their belief about the Phillips curve. Furthermore, in order to estimate a perceived Phillips curve of the average forecaster, equation (11) illustrates that given correlations of survey data on inflation and unemployment, it will be necessary to explicitly model and estimate the degrees of information frictions.

The next section presents the empirical model used in this paper, which contains a more realistic forecasting model for inflation and unemployment, following a trend-cycle decomposition of inflation and unemployment combined with forecasting smoothing equations similar to the ones presented above.

3 The Empirical Model

The joint law of motion of real-time data and survey expectations of inflation and unemployment consists of three main components. First, a joint law of motion of inflation and unemployment, which reflects the forecasting model, i.e. the perceived law of motion, of the average professional forecaster. Second, a law of motion for survey expectations, which, based on the forecaster's forecasting model, generates now-casts of inflation and unemployment. The last component is a mapping of current states and now-casts of trend and cycles of inflation and unemployment to real-time inflation and unemployment data and to the available term structure of inflation and unemployment survey forecasts. The empirical model extends Mertens and Nason (2017) by giving a role to unemployment survey forecasts in explaining inflation survey dynamics and by setting up a correspondingly richer structure of information frictions.

The perceived law of motion of inflation and unemployment follows a bi-variate trend-cycle decomposition following Chan et al. (2016). Here the inflation and unemployment trend both follow a random walk with stochastic volatility for inflation trend innovations and constant volatility for unemployment. The unemployment cycle follows an AR(2) process with constant volatility. The inflation cycle component incorporates one lag of the inflation gap, a contemporaneous unemployment gap, and stochastic volatility in the innovation term. The coefficients on the inflation gap lag, and the contemporaneous unemployment gap, i.e. the Phillips curve slope, are both allowed to be time-varying, following a random walk. The trend-cycle decomposition can be summarized in the following form

$$z_{t+1} = B_{t+1} z_t + C_{t+1} \nu_{t+1} \qquad \nu_{t+1}, \sim \mathcal{N}(0, I), \tag{12}$$

where B_{t+1}, and C_{t+1} contain all time-varying parameters, stochastic volatility components, and constant coefficients. z_{t+1} contains trend and cycle components of inflation and unemployment. Trend and cycles z_t link to states of inflation and unemployment, i.e. $y_t = (\pi_t, u_t)'$, via

$$y_t = G z_t. \tag{13}$$ [9]

At this point, I want to reemphasize, that this econometric model represents the average forecaster's **perceived** law of motion of the economy. Section 3.1 will provide a discussion of the main components of the perceived law of motion as they are relevant for the result section.[10]

Following Coibion and Gorodnichenko (2015), and Mertens and Nason (2017) who add time-variation to forecast rigidities, professional forecasters compute their now-casts of states z_t following a forecast smoothing rule based on imperfect information frameworks,

[9] Note, that a comparison of this specification to the empirical model in Section 2 would call for the addition of a $\cdot^{PLM}$ superscript on all coefficient matrices. This will be omitted for the sake of readability.

[10] Section A.2 provides a detailed description of the bi-variate trend-cycle decomposition used.

such as noisy or sticky information, given by

$$F_{t+1}z_{t+1} = \Lambda^z_{t+1}F_t z_{t+1} + \left(I - \Lambda^z_{t+1}\right) z_{t+1}. \tag{14}$$

$F_t z_t$ represents now-casts of all trend and cycle components z_t. The matrix Λ^z_t keeps track of the time-varying degrees of information frictions in inflation and unemployment survey forecasts. Therefore the average forecaster's now-casts are weighted averages of her past forecasts and newly incoming information. In the empirical model, the 'weighting' matrix Λ^z_t incorporates two time-varying parameters λ^π_t and λ^u_t on its diagonal. This restriction aims at minimizing the amount of time-varying parameters to be estimated, while still acknowledging the fact that the degrees of information frictions of different variables can differ.[11] Both parameters are allowed to follow random walks constraint to the interval $[0, 1]$. In principle, the smaller these parameters are, the more frequently forecasters update their survey forecasts. As already mentioned, the forecast smoothing equation (14) is consistent with a noisy information framework, where forecasters receive noisy signals about the present states of inflation and unemployment. Given such a signal extraction problem the forecast smoothing rule (14) results naturally from the application of the Kalman filter.[12]

Taking (12) as the average forecaster's forecasting model, it follows that given now-casts $F_{t+1}z_{t+1}$ the average forecaster computes a one-step ahead forecast by using the coefficient matrix B_{t+1}. At this point time-variation in parameters creates a problem. Higher horizon forecasts $F_{t+1}z_{t+h}$ would have to depend on predictions of future values for the coefficient matrix B_{t+1}. To avoid such an additional prediction step, I make use of the augmented utility model (AUM).[13] The AUM implies that, when forecasters compute forecasts or filtered states at time $t+1$, they also fix their estimates of coefficient matrices B_{t+1}, and C_{t+1} at time t+1. Therefore instead of B_{t+1} and C_{t+1}, I write $B_{t+1|t+1}$ or $C_{t+1|t+1}$, respectively.

[11] A fully consistent noisy information specification would imply a fully occupied matrix Λ (25 elements) with all components being non-linear functions of the forecasting models' state covariances and noise variances from forecasters' signal extraction problem. Modeling and estimating such a structure in detail might be a fruitful avenue for future research.

[12] For more details and a derivation of the forecast smoothing equation see Section A.3 in the appendix.

[13] This is a common assumption in the literature, see for example Kreps (1998), Cogley and Sargent (2008), Cogley and Sbordone (2008), Mertens and Nason (2017). More details can be found in Section A.1 in the appendix.

Plugging in (12) into (14) and making use of the AUM yields

$$F_{t+1}z_{t+1} = \Lambda^z_{t+1}B_{t+1|t+1}F_t z_t + \left(I - \Lambda^z_{t+1}\right)B_{t+1|t+1}z_t + \left(I - \Lambda^z_{t+1}\right)C_{t+1|t+1}\nu_{t+1}. \quad (15)$$

Stacking both the forecasting model (12) and the forecast smoothing rule (15) results in a joint description of inflation and unemployment data and its now-casts

$$\begin{pmatrix} z_{t+1} \\ F_{t+1}z_{t+1} \end{pmatrix} = \begin{pmatrix} B_{t+1|t+1} & 0 \\ \left(I - \Lambda^z_{t+1}\right)B_{t+1|t+1} & \Lambda^z_{t+1}B_{t+1|t+1} \end{pmatrix} \begin{pmatrix} z_t \\ F_t z_t \end{pmatrix} + \begin{pmatrix} C_{t+1|t+1} \\ \left(I - \Lambda^z_{t+1}\right)C_{t+1|t+1} \end{pmatrix} \nu_{t+1}. \quad (16)$$

To complete the setup of the empirical model, it remains to link latent states to observables. To that purpose now-casts $F_t z_t$ and coefficient matrix $B_{t|t}$ are used to map to the term structure of inflation and unemployment survey forecasts via

$$F_t^{\text{survey}} y_{t+h} = G B^h_{t|t} F_t z_t + \zeta_{t,h}, \qquad \zeta_{t,h} \sim \mathcal{N}\left(0, \sigma^2_{\zeta,h}\right). \quad (17)$$

where $F_t^{\text{survey}} y_{t+h}$ are observed survey measures for time horizons $h = 1, \ldots, H$ for inflation and unemployment, and $\zeta_{t,h}$ represent measurements errors. Taken together with (13) these define the state-space system's observation equation, where (16) taken together with laws of motion for time-varying parameters define the state equations. A more detailed description of the joint law of motion of inflation and unemployment real-time data and survey expectations can be found in Section A.4 in the appendix.

3.1 The Perceived Phillips Curve Slope

Using the econometric model above, I proceed to provide some details about the empirical Phillips curve specification in order to gain some intuition about the relevant dynamics. In the following, I drop the conditional time subscript. Taking the model component borrowed from Chan et al. (2016), the Phillips curve in the forecasting model (12) is given by a short-run Phillips curve defined on the cycle components of inflation and unemployment

$$\pi^{cycle}_{t+1} = \rho_{t+1}\pi^{cycle}_t + \gamma_{t+1}u^{cycle}_{t+1} + \sigma^{cycle,\pi}_{t+1}\epsilon^{\pi}_{t+1}, \quad (18)$$

where ρ_{t+1}, γ_{t+1}, and $\sigma_{t+1}^{cycle,\pi}$ are components of the matrices B_{t+1} and C_{t+1} introduced in the last section. ρ_{t+1} tracks the persistence of the inflation gap over time and $\sigma_{t+1}^{cycle,\pi}$ stochastic volatility in the inflation gap. The parameter of interest in this paper is the time-varying Phillips curve slope γ_{t+1}, which contemporaneously relates changes in the unemployment gap to the inflation gap. When using such a specification to compute inflation forecasts a specification of the law of motion of unemployment is needed as given by

$$u_{t+1}^{cycle} = \theta_1 u_t^{cycle} + \theta_2 u_{t-1}^{cycle} + \sigma^{cycle,u} \epsilon_{t+1}^{u}. \tag{19}$$

Estimates of γ_{t+1} therefore provide a measure of the perceived Phillips curve slope of the average professional forecasters, as far as it is used in her forecasting model. In the following I set

$$\gamma_t^{PLM} := \gamma_t, \tag{20}$$

to reemphasize the parameter's role as a component of the forecaster's perceived law of motion. Similar to Section 2, forecast smoothing equations for cycle components are given by

$$F_{t+1}\pi_{t+1}^{cycle} = \lambda_{t+1}^{\pi} F_t \pi_{t+1}^{cycle} + \left(1 - \lambda_{t+1}^{\pi}\right) \pi_{t+1}^{cycle} \tag{21}$$

$$F_{t+1}u_{t+1}^{cycle} = \lambda_{t+1}^{u} F_t u_{t+1}^{cycle} + \left(1 - \lambda_{t+1}^{u}\right) u_{t+1}^{cycle}, \tag{22}$$

where each now-cast is a weighted average of the past forecast and current data.

Again, let us suppose that an econometrician gets his hands on some present and past survey data and wants to compute the correlation of inflation and unemployment now-casts conditional on a surprise change in the unemployment gap in period t.[14]. Similar to Section 2, she would arrive at

$$\gamma_t^{SPF} := Corr\left(F_t \pi_t^{cycle}, F_t u_t^{cycle} | \mathcal{I}_{t-1}\right) = \frac{1 - \lambda_t^{\pi}}{1 - \lambda_t^{u}} \gamma_t^{PLM}, \tag{23}$$

where $\mathcal{I}_{t-1}$ is the information set at time $t-1$ including forecasts and data from period $t-1$. γ_t^{SPF} might be given different interpretations. First, it measures the response of inflation now-casts to unemployment now-cast changes following a contemporaneous

[14]A detailed derivation of the conditional correlation can be found in Section A.5 in the appendix.

surprise change in the unemployment gap. Second, it shows the 'bias' due to information frictions, when attempting to interpret γ_t^{SPF} as some 'belief' of professional forecasters concerning the Phillips curve slope. The intuition behind this results holds similarly to Section 2 with the difference being, that the deviation of γ_t^{PLM} to γ_t^{SPF} now depends on time-variation in the difference of degrees of information frictions in inflation and unemployment.

4 Data and Estimation Strategy

4.1 Data

I use survey data and real-time data from the Survey of Professional Forecasters and Real-Time Data Set for Macroeconomists, respectively, as released by the Philadelphia Federal Reserve Bank spanning 1968-Q4 to 2018-Q3. Professional forecasters provide a now-cast and 1 to 4 quarter ahead inflation and unemployment forecasts since 1969 in a quarterly frequency. I make use of forecasts for the GDP deflator and the unemployment rate. One word of caution concerning the timing of survey data. Following the documentation[15] of the Survey of Professional Forecasters by the Philadelphia Federal Reserve Bank, surveys are usually conducted in between real-time data releases. So when professional forecasters provide now-casts, the information reflected in the forecasters' responses are more akin to one quarter ahead forecasts. I will therefore treat the now-cast as a one-step-ahead forecast, resulting in a five-period expectation term structure.

I decided to use professional forecasters' survey expectations for several reasons. First, as I attempt to estimate a model with time-variation in parameters and to allow for significant, economically relevant changes in these parameters, the need for sufficiently long times series for inflation and unemployment data and survey expectations arises. The Philadelphia FED's time series on survey responses on the GDP deflator and unemployment start in 1969, which allows for the evaluation of significant events in U.S. economic history such as both oil crises, the Great Moderation, the Great Recession, and its aftermath. Second, concerning the relevance of professional forecasters' expectations for firms'

[15] https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en

price-setting decisions, Carroll (2003) establishes that there indeed exists an information diffusion process from professional forecasters to firms and households. In addition, forecasters have the resources and professional interest - towards their customers - to provide reliable indicators of macroeconomic outcomes. Their perception of uncertainty around states of the economy can therefore be seen as a lower bound for the uncertainty of firms and households. Last, as established by Ang et al. (2007), median survey responses by professional forecasters of inflation in the U.S. are strong predictors of actual future inflation. That indicates that forecasters do seem to have information which cannot be found in other macroeconomic or financial variables.

4.2 Estimation Strategy

The state-space model is non-linear in latent states and parameters.[16] To make the filter problem more efficient, I make use of the special conditionally linear Gaussian structure of the problem. Conditional on all non-linear states,[17] the state-space model reduces to a conditional linear Gaussian model. This allows the application of Rao-Blackwellized filtering techniques, which result in large gains in efficiency and in a decrease in the number of particles needed to arrive at reasonable state estimates. After marginalizing out non-linear states, the remaining filter problem can be solved by the application of a standard Kalman filter. More specifically, I apply the Rao-Blackwellized auxiliary particle filter (RB-APF) as in Creal (2012). Conditional on K particle draws[18] of linear and non-linear states, a Kalman prediction step computes marginal log-likelihoods, with which all particles can then be re-weighted according to their likelihood. After re-sampling, which mostly duplicates 'good' particles with a larger weight given by its likelihood, the RB-APF propagates non-linear states using its random walk specifications. Hereafter, sufficient statistics on linear-states are updated using a full Kalman filter step, based on

[16]The full model state-space model specification, presented in the appendix Section A.4, consists of state transitions (51) for states and perceived states, and the observation equation (52).

[17]Time-varying parameters in matrices $\mathcal{B}_t$ and $\mathcal{C}_t$ are given by

$$\mathcal{V}_t = \left\{ \rho_t, \gamma_t, \lambda_t^{\pi}, \lambda_t^{u}, h_t^{trend,\pi}, h_t^{cycle,\pi} \right\}, \tag{24}$$

where ρ_t is the inflation gap persistence, γ_t is the Phillips curve slope, λ_t^{π} is the degree of information frictions in inflation, λ_t^{u} is the degree of information friction in unemployment, $h_t^{trend,\pi}$ is stochastic volatility in trend inflation, and $h_t^{cycle,\pi}$ stochastic volatility in gap inflation.

[18]I choose the number of particles to be $K = 150.000$.

new draws of non-linear states. This approach is repeated for all $t = 1, \ldots, T$. Details on the algorithm can be found in the appendix Section B.1.1. Measurement error variances for all inflation and unemployment survey forecasts are computed sequentially using Particle Learning (PL) approaches by Carvalho et al. (2010) and Lopes and Tsay (2011). The PL adds draws of variance coefficients from an inverse gamma distribution and of their respective sufficient statistics to the particle set. The sufficient statistics are propagated each period using a closed-form conditional posterior for the inverse gamma distribution, as new information arrives in period t. This is similar to Mertens and Nason (2017), who provide a RB-APF with PL to estimate the full posterior of states and parameters of their model. However, in this paper, in order to reduce the complexity of the estimation procedure, all constant parameters besides measurement error variances are calibrated following the literature or estimated using maximum likelihood in a separate state-space model. Further details can be found in Section B.3 in the appendix.

5 Results

The algorithm presented in the last section provides filtered estimates for linear and non-linear states, and probability bands at the 16th and 84th quantile. In the following, I will first conduct a model comparison exercise to elicit if a Phillips curve in the econometric model performs well in explaining aggregates and survey responses. Next, I present the perceived Phillips curve slope, discuss its evolution, and also present the estimates for both parameters governing information frictions. After that, I provide estimates on the difference in information frictions and on the correlation of inflation and unemployment now-casts following a surprise change in the unemployment gap, as introduced in Section 3.1. I will close this section with a discussion on the co-movement of the perceived Phillips curve slope with the degrees of information frictions in inflation and unemployment. Non-linear state estimates presented in this section start in 1971-Q1 to eliminate the effect of diffuse priors used in implementing the Kalman filter.

5.1 Model Comparison

The question if the average professional forecaster has a Phillips curve in mind when reporting survey forecasts can also be cast in the following way: Does the joint law of motion of inflation and unemployment survey data perform better when the forecasting model incorporates a link between inflation and unemployment? Such a question can be answered in terms of the model fit between a model with and without a Phillips curve slope. Therefore in this section, I evaluate the relevance of embedding a Phillips curve slope in the average forecaster's forecasting model. Models will be evaluated using marginal data densities (MDD) computed over the period 1968-Q4 - 2018-Q3. MDDs are commonly used in Bayesian model selection. One advantage of the sequential nature of the RB-APF is the direct availability of MDDs for each time t. To compare models, I compute Log Bayes Factors given by

$$\text{Log Bayes Factor} = \log MDD_1 - \log MDD_2. \tag{25}$$

Log-Bayes factors above ten usually represent strong support for model 1 against model 2. As a reminder, the baseline model is following Chan et al. (2016), including stochastic volatility in the inflation trend and inflation gap, time-variation in inflation gap persistence, and the Phillips curve slope. Unemployment trend and cycle volatility and both lags of unemployment gap persistence are assumed to be constant. The comparison is performed using the same model, in which the Phillips curve slope parameter is set to zero.

(PLM) model	
$\gamma_t^{PLM} \neq 0$	-801.79
$\gamma_t^{PLM} = 0$	-842.04
log Bayes Factor	40.25

Table 1: log MDDs

As is apparent from Table 1 the model fit criterion is strongly favoring a joint law of motion which includes a Phillips curve slope.

5.2 Average Forecaster's Phillips Curve and Information Frictions

Figure 1 shows the average professional forecaster's perceived Phillips curve slope. Being flat during the 1970s, it became very steep shortly before the onset of the Great Moderation. Afterward, the slope coefficient decreased only slightly in absolute value until the middle of the 1990s. Then, an accelerated flattening took place until the 2000s. From this time forward, the slope coefficient remained at approximately -0.3 to -0.1. Only at some periods is the zero line contained in the probability bands. Note, that I did not constrain the slope coefficient a priori to the interval $[-1, 0]$ within the estimation strategy. This is further evidence that not only does the average professional forecaster have a Phillips curve in mind when reporting survey forecasts, but that its sign is consistent with macroeconomic theory. Qualitatively, the result presented here is similar to results for the slope of the New Keynesian Phillips curve based on the output gap by Kuttner and Robinson (2010). Similar to Figure 1, the authors find that the Phillip curve slope parameter becomes large in absolute values towards the 1980s, and only started to decrease during the 1990s. Multiple possible explanations have been put forward to explain this observation, such as the effects of globalization, data revisions, and others,[19] which might similarly apply to agent's perception concerning the Phillips curve.

Figure 1: γ_t^{PLM} and 14 - 86 % probability bands

[19]For a more detailed discussion see Kuttner and Robinson (2010).

One possible explanation, on which this paper can shed some light, is that the properties of the expectation formation process changed over time. For example, Lansing (2009) finds a downward-drift in the perceived signal-to-noise for inflation, which indicates a reduced likelihood of a permanent shift in the inflation trend, reflecting increased monetary policy credibility. Furthermore, Lansing (2009) argues that within his consistent expectations framework, a decline in the Phillips curve slope parameter should be accompanied by a decline in the perceived signal-to-noise ratio. Within the context of this paper, this implies that a flattening Phillips curve should be accompanied by increased stickiness of inflation forecast, i.e. λ^{π} approaching one. The analysis in this paper can provide independent evidence to that effect, as I am estimating the perceived Phillips curve slope and a measure of the forecaster's signal-to-noise ratio of inflation in a reduced-form empirical model.[20]

(a) λ_t^{π} with 14 - 86 % probability bands (b) λ_t^{u} with 14 - 86 % probability bands

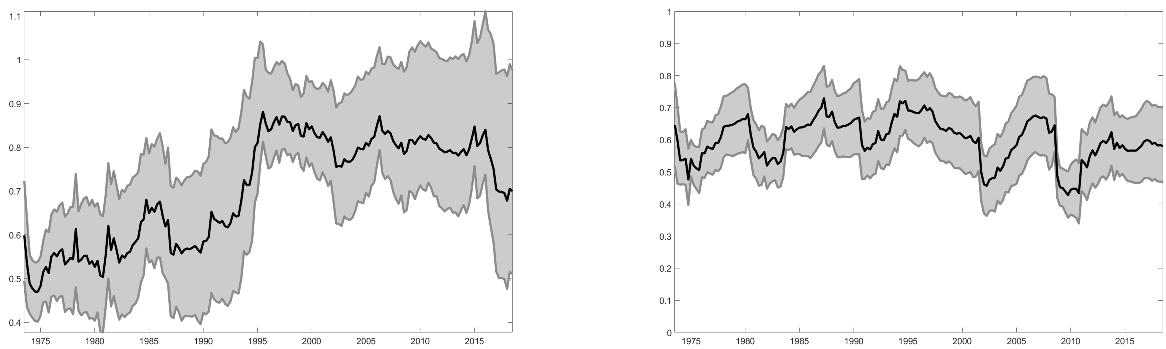

Figure 2

Figure 2 shows the degree of information frictions for inflation in the left plot and the degree of information frictions for unemployment in the right plot. Inflation forecasts seem to have been updated more regularly in the past than today. During the Great Moderation and especially during the first half of the 1990s, the updating frequency decreases substantially. This is qualitatively in line with findings in Mertens and Nason (2017), albeit their estimate of the inflation information friction parameter is smaller at the beginning of the sample. Furthermore, the degree of information friction in inflation

[20]The signal-to-noise ratio here being closely related to the degree of information friction in inflation. In a noisy information framework, the parameter λ^{π} is a measure of the Kalman gain in the forecaster's signal extraction problem.

seems to be proceeding the flattening of the Phillips curve slope by around half a decade. The delay could be rationalized by the time it takes for information to fully disseminate from professional forecasters to all firms and household.[21] Section 5.4 discusses the state-dependence of the perceived Phillips curve in more detail. Concerning unemployment information frictions, it seems that the updating frequency of unemployment forecasts remains relatively stable over longer time periods.

Nevertheless, the higher frequency fluctuations seem to be of interest here. As we can see in Figure 3, increased frequencies of unemployment forecast updating coincide with NBER recessions. This is not very surprising as unemployment usually increases suddenly and sharply during recessions, leading to more frequent updating of unemployment expectations. Note also that the Great Recession did trigger more frequent updating of inflation forecasts, but rather that of unemployment forecasts.

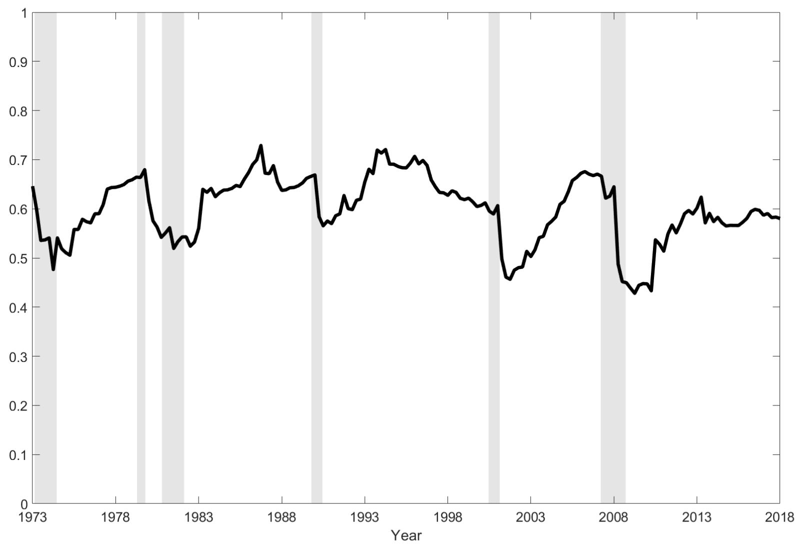

Figure 3: λ_t^u and NBER recession bands

5.3 Information Frictions Bias

The purpose of this section is two-fold. First, I provide estimates of the correlation of inflation and unemployment now-casts following a surprise change in the unemployment gap, which an econometrician might arrive at when using survey data. This aims at providing evidence towards the 'bias' information frictions induce when the researcher attempts to

[21] Carroll (2003) argues that it might take some time for news of changes macroeconomic circumstances to reach all agents in the economy.

learn about forecaster's perceived model of the economy. Second, the correlation can be regarded as a measure of the difference in information frictions, i.e. the difference in the updating frequency of inflation and unemployment over time.

In Section 3.1, I presented the correlation of inflation and unemployment survey nowcasts conditional on a surprise change in the unemployment gap at time t given by

$$\gamma_t^{SPF} := Corr\left(F_t\pi_t^{cycle}, F_t u_t^{cycle} | \mathcal{I}_{t-1}\right) = \frac{1-\lambda_t^{\pi}}{1-\lambda_t^{u}}\gamma_t^{PLM}. \tag{26}$$

In the following, I will refer to γ_t^{SPF} as the 'survey data slope' and to γ_t^{PLM} as the 'perceived slope'.

Figure 4a shows the perceived slope with a dashed black line and the survey data slope with a solid black line. Figure 4b shows the relative difference of both. It is interesting to note that Figure 4b also has an interpretation as a measure of the difference in information frictions given by

$$\frac{\gamma^{SPF}-\gamma^{PLM}}{\gamma^{PLM}} = \frac{1-\lambda_t^{\pi}}{1-\lambda_t^{u}} - 1. \tag{27}$$

Values larger than zero indicate that unemployment surveys are updated less quickly than inflation surveys, values lower than zero imply the opposite. Observe that the difference in information frictions is time-varying and that its sign is changing over time.

(a) γ^{PLM} (dashed line) and γ^{SPF} (solid line)

(b) $\frac{\gamma^{SPF}-\gamma^{PLM}}{\gamma^{PLM}}$

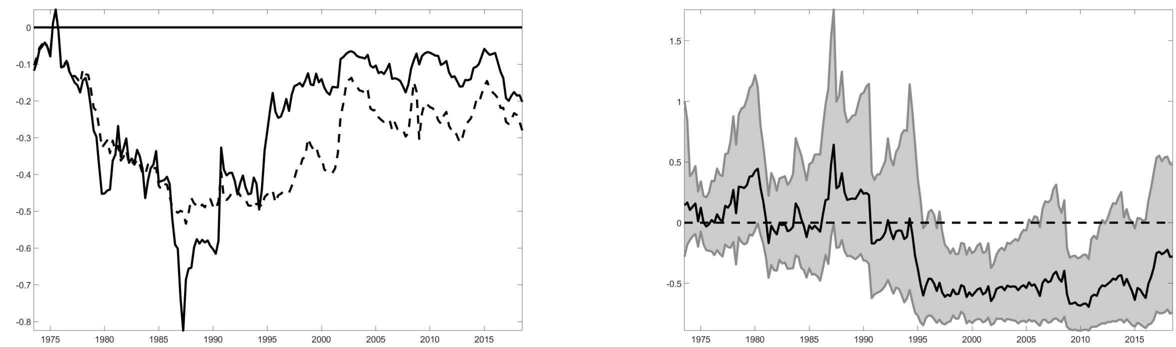

Figure 4

Both slope parameters appear to be quite similar until around the start of the Great Moderation. Shortly after, the survey data Phillips curve steepens until the deviation to the perceived slope amounts to around 60% in absolute values. This is due to unemployment forecasts being updated less quickly than inflation forecasts during this period. This dynamic quickly changes as the frequency of inflation forecast updating increases rapidly during the 1990s. Judging from Figure 4b the deviation in the 1980s is barely significant, whereas the deviation starting in the 1990s, remains significantly below zero for an extended period. One take-away from this figure is that the conditional correlation of inflation and unemployment surveys consistently underestimates the perceived slope of the average professional forecaster by about 50% to 60%. This implies that when we are interested in the actual strength of the inflation-unemployment trade-off as agents perceive it, measures for their degrees of uncertainty about the current state of the economy have to be taken into account.

Since Figure 4b can also be interpreted as a relative measure of the difference between forecast updating frequencies in inflation and unemployment, it is useful as a test of the properties of information frictions. One prediction of standard models of sticky information, as in Mankiw and Reis (2002), is that the arrival rate of new information is the same across all variables. As parameters $\lambda^{\pi}, \lambda^{u}$ can also be interpreted as arrival probabilities of information about inflation or unemployment,[22] here, we can see that this prediction does not hold over the whole sample period. However, this observation is consistent with models of noisy information, as parameters governing information frictions can be state-dependent.

5.4 State-dependency of the Phillips Curve Slope

Following up on the discussion in Section 5.2 concerning the lagged relationship between the perceived Phillips curve slope and the degree of information frictions in inflation, this section aims to zoom in on the time-variation in the perceived Phillips curve slope of the average professional forecaster. More specifically, I am interested in the potential co-movement of the Phillips curve slope with parameters governing information frictions, i.e. λ^{π} and λ^{u}. To guide this discussion, it is helpful to look to the theoretical literature, which might provide predictions concerning co-movement in states and parameters, that

[22] In models of sticky information, the 'information friction' arises from the exogenously given arrival rate of information. After new information is revealed, the agents fully incorporate that information.

can be tested using reduced-form empirical models. The period considered in this exercise starts in 1979-Q1 to focus especially on the flattening of the Phillips curve since the 1980s and 1990s.

5.4.1 Candidate Explanations

First, in models of costly price adjustments (sticky-prices), as presented for example in Ball et al. (1988) and Ball and Mazumder (2011), when permanent inflation and inflation variance is high, firms change their prices more frequently compared to low inflation trend and volatility environments. This implies that for any given level of nominal aggregate demand, the Phillips curve slope would be flatter under low inflation than under high inflation. This regularity is economically intuitive. As argued by Mazumder (2012), if inflation is high and volatile, large swings in the price level occur, which would make it costly to hold prices constant. When inflation is low and less volatile, there is little need for frequent price adjustments. Ball and Mazumder (2011) find that this is indeed the case in the U.S. for the period starting in late 1980 through 2011.

The second candidate explanation will be based on models of imperfect information, or more specifically, models of rational inattention as initiated by Sims (2003). In rational inattention models, agents incur a cost when attempting to pay attention to specific pieces of information. The scarcity of attention will lead to an optimal joint choice of actions and the amount of attention. When paying attention is costly, it is optimal for agents to not fully incorporate new information in their information set and compute actions based on that 'imperfect' piece of information. Forecasters' attention can be tracked by their perceived signal-to-noise ratio. For example, Maćkowiak and Wiederholt (2009) and Maćkowiak and Wiederholt (2015) show that the strength of the reaction of price-setting of firms in response to changes in aggregate conditions diminishes, as firms pay less attention to macroeconomic conditions. Therefore the perceived Phillips curve would flatten, as forecasters' attention decreases. It is worth noticing that the dependence of the Phillips curve slope on some measure of the degree of bounded rationality is not exclusive to models of rational inattention. Take, for example, the Sticky-Information Phillips Curve in Mankiw and Reis (2002), the Phillips curve in the behavioral New Keynesian model by Gabaix (2016), or as discussed in the last section the bounded rationality model by

Lansing (2009). I chose to use the rational inattention perspective as it gives a behavioral dimension to the parameters of information frictions. When attention is costly, agents choose the optimal attention allocation to inflation and unemployment, implying time-variation in the signal-to-noise ratio as economic conditions change.

5.4.2 Regression Results

In the following, instead of λ_t^π, and λ_t^u, I set

$$\kappa_t^\pi := -\frac{1}{2}\log\left(\lambda_t^\pi\right) \tag{28}$$

$$\kappa_t^u := -\frac{1}{2}\log\left(\lambda_t^u\right), \tag{29}$$

calling κ_t^π and κ_t^u forecaster's attention to inflation and unemployment, respectively. This choice is motivated by the literature on rational inattention, where attention tracks the mutual information concerning the state of interest and the respective signal. For example, as signals on inflation become more informative, increasing the signal-to-noise ratio, survey forecasts are updated more quickly, implying a lower degree of information frictions, and therefore higher attention to inflation. For a short exposition of rational inattention and a derivation of these transformations see Section C.2 in the appendix. Further, I set $\kappa_t = \kappa_t^\pi + \kappa_t^u$ as a joint measure of attention.

Summarizing both strands of literature in regression model 1, the perceived Phillips curve slope is regressed on traditional factors of models of costly price adjustment such as trend inflation and the sum of permanent and transitory inflation volatility, but also on factors relevant to imperfect information models such as attention κ_t as defined above.

Regression model 1:

$$\gamma_t^{PLM} = \alpha_0 + \underbrace{\alpha_1 \pi_t^{trend} + \alpha_2 \sigma_t^{trans+perm,\pi}}_{\text{costly price adjustments}} + \underbrace{\beta\kappa_t}_{\text{Attention}} + \epsilon_t \tag{30}$$

This allows me to estimate the relevance of attention on the perceived slope while holding inflation trend and volatility constant, and vice versa.

Regression model 2:

In regression model 2, I split up the joint attention parameter into its components, attention to inflation and attention to unemployment.

$$\gamma_t^{PLM} = \alpha_0 + \underbrace{\alpha_1 \pi_t^{trend} + \alpha_2 \sigma_t^{trans+perm,\pi}}_{\text{costly price adjustments}} + \underbrace{\beta_1 \kappa_t^{\pi}}_{\text{Attention to inflation}} + \underbrace{\beta_2 \kappa_t^{u}}_{\text{Attention to unemployment}} + \epsilon_t \tag{31}$$

Table 2 shows results of both regression models in columns 1 and 2, respectively. Column 3 shows regression estimates of model 1 with standardized regressors, column 4 similarly for regression model 2.

			Standardized Regressors	
γ_t^{PLM}	(1)	(2)	(3)	(4)
const	−0.35 (0.06)	−0.49 *** (0.16)	0.00*** (0.02)	0.00*** (0.12)
π_t^{trend}	−0.02 *** (0.01)	0.02** (0.01)	−0.43 * (0.25)	0.37 (0.23)
$\sigma_t^{trend,\pi} + \sigma_t^{cycle,\pi}$	0.015 (0.017)	−0.014*** (0.013)	0.07 (0.17)	−0.14 (0.12)
κ_t	0.24 (0.19)		0.28** (0.13)	
κ_t^{π}		−0.58 *** (0.21)		−0.50 ** (0.19)
κ_t^{u}		0.95*** (0.21)		0.56*** (0.13)

Table 2: 1979:Q1–2018:Q3, with HAC standard errors, bandwidth 4 (Bartlett kernel)

Results in column 1 support the traditional sticky-price view that the inflation trend would be negatively correlated with the Phillips curve slope, as an increase in trend inflation results in a stepper (more negative) slope. Joint attention does not seem to play a role, in this respect. Using standardized regressors in column 3, the inflation trend turns out to be the dominant factor in explaining variation in the perceived Phillips curve slope. This changes after attention on inflation and unemployment are taken into account separately. The sign on the inflation trend reverses and becomes insignificant in the standardized specification. Pointing towards the relevance of models of imperfect information, attention to inflation is now negatively and significantly correlated with the perceived Phillips curve slope. Interestingly, attention to unemployment seems to be similarly relevant in explaining the perceived slope, but it is positively correlated with the perceived slope coefficient.

Section 5.2 showed that the flattening of the Phillips curve was preceded by an increase in the degree of information friction in inflation, i.e. a fall in attention to inflation. To evaluate these dynamics quantitatively, I estimate the following specification, where the Phillips curve slope t is regressed on past factors of costly price adjustment and attention.

Regression model 3:

$$\begin{aligned}\gamma_t^{PLM} = \alpha_0 + &\underbrace{\alpha_1 \pi_{t-h}^{trend} + \alpha_2 \sigma_{t-h}^{trans+perm,\pi}}_{\text{costly price adjustments}} + \\ &\underbrace{\beta_1 \kappa_{t-h}^{\pi}}_{\text{Attention to inflation}} + \underbrace{\beta_2 \kappa_{t-h}^{u}}_{\text{Attention to unemployment}} + \epsilon_t \end{aligned} \tag{32}$$

(a) (b)

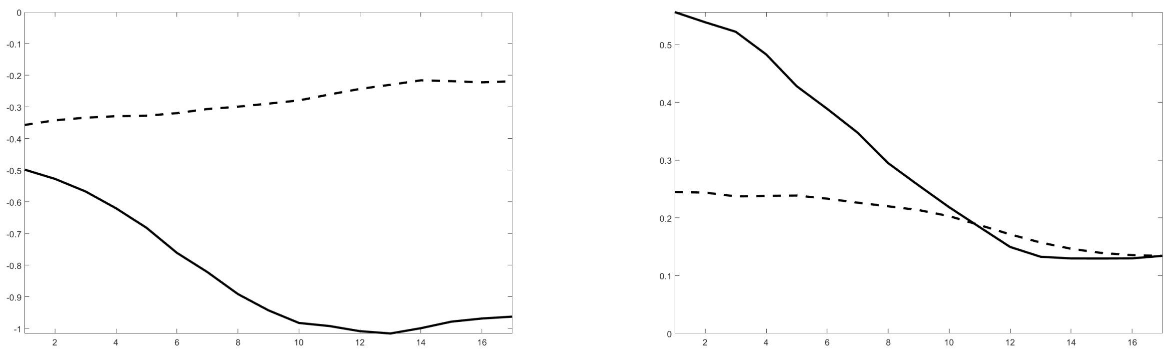

Figure 5: The solid lines represent regression coefficients of attention to inflation (left) and attention to unemployment (right) for each h (x-axis) in regression model 3. Dashed lines represent t-statistics at a 5% significance level.

Figure 5a shows the coefficients on attention to inflation re-estimated for each $h = 1, \ldots, 16$, together with t-statistics at a 95% confidence level. Similarly, Figure 5b shows the same for attention to unemployment. The coefficient on attention to inflation increases steadily in absolute value until $h = 12$, reflecting the delay observed in Section 5.2. At the same time, the coefficient of unemployment attention decreases and becomes insignificant at around the same time horizon. This implies the positive correlation observed in Table 2 acts on a shorter time interval on the perceived Phillips curve slope, than attention to inflation does.

The negative correlation of attention to inflation with the perceived Phillips curve slope is consistent with predictions of imperfect information models. A similar result on the attention to inflation can also be found in Blanchard et al. (2015). The authors provide evidence for a flattening Phillips curve slope and more stable anchoring of short-run inflation expectations to long-run expectations. As κ^{π} in this paper also measures the frequency with which inflation forecasts are updated, these concepts are closely related.

The positive correlation of the perceives slope coefficient, and attention to unemployment remains puzzling. A possible explanation is related to (misperception of) hysteresis effects in unemployment. Forecasters might believe that the unemployment trend is increasing less sharply during a recession than it actually does. This implies that their perceived unemployment gaps widen, while their perceived inflation gaps remain the same, leading to a flattening perceived Phillips curve.[23] Since forecasters increase their attention to unemployment during a recession, reduced-form evidence would show a positive correlation of the perceived slope coefficient and attention to unemployment. Figure 5b supports such an interpretation, as these effects should only appear on business cycle frequencies.

6 Conclusion

This paper shows that the average professional forecaster indeed has a Phillips curve in mind when reporting survey forecasts. However, the presence of frictions in the updating process of forecasters' information sets results in a sizable bias to forecasters' perceived correlation coefficients. I provided a closed-form representation of that deviation and examined the bias for forecasters' Phillips curve slope. However, such an analysis is not

[23]For example, if they neither under or overestimate the inflation trend.

only limited to the application on the Phillips curve. In principle, for any macroeconomic relationship which links at least two distinct aggregates such a bias would appear, as long as the degrees of information frictions in both variables are sufficiently different. Re-evaluation of the Euler equation, Okun's law, or of the Taylor rule could pose interesting avenues for future research. The main difference to the existing literature that studies the perceived Phillips curve empirically lies in the interpretation of model-consistency. The presence of information frictions makes it necessary to carefully re-evaluate what we mean when we ask if survey expectations are consistent with standard macroeconomic concepts. These concepts have their origins in an environment where information is perfectly available to all agents in the economy. However, deviations from full information rational expectation models can be sizable, which calls for the explicit consideration of information frictions, however, modeled, when analyzing survey responses for macroeconomic aggregates.

As shown in this paper, the correlation of inflation and unemployment survey now-casts can be linked to the inflation and unemployment correlation in the forecaster's perceived model of the economy, by taking the effects of information frictions into account. However, even after this step, the last section showed that there remains a sizable co-movement between the perceived slope parameter and parameters governing information frictions. This calls for attempts to set up econometric models of forecasters' forecasting model and their expectation formation process based on 'deep structural' parameters, which can explain the variation in both model components.

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A Derivations and Model definition

A.1 Time-variation in State-Space Dynamics and Noisy Information - AUM

Time-variation, specified as parameters following random walk specifications, in the perceived law of motion of forecasters, poses some problems when we want to specify the forecasting behavior of the average survey forecaster. Assuming agents are fully aware of the model, including random walk specifications, invoking the LIE to compute expectations is not possible anymore.[24] Therefore computing higher horizon forecasts becomes a non-linear problem even if, conditionally on time-varying parameters, the model is linear. To work around this caveat, it has become common in the empirical literature [25] to assume that agents follow the anticipated utility model (AUM). Here, agents do realize that the underlying model does include time-variation and are therefore aware that the present state can be described by the history of time-variation in parameters. However, and this is key, concerning the future they treat coefficient as being constant. Therefore when forecasting states at $t + h$ in t, parameters will be held fixed at t. Concerning the plausibility of that assumption, one can either assume, that agents follow some AUM, or consider it as an approximation to the 'true' non-linear Bayesian updating model. Cogley and Sargent (2008) provide evidence that if certainty equivalence holds the AUM is able to sufficiently approximate full Bayesian updating models.

Within the context of this paper, invoking the AUM, I can assume that

$$F_t z_{t+h} = B_{t|t}^h F_t z_t, \tag{33}$$

where $B_{t|t}$ is the present filtered state of B_t.

[24] As is the application of the Kalman-Filter, since the model is not linear.

[25] See Kreps (1998), Cogley and Sargent (2008), Cogley and Sbordone (2008), Mertens and Nason (2017)

A.2 Forecasters' Model

This section lays out in detail the econometric specification used as the perceived law of motion of inflation and unemployment. The setup is closely following Chan et al. (2016). For simplicities sake, I omit conditional time subscripts.

Forecasters decompose inflation π_t and unemployment u_t into trend and cycle (gap) components via

$$\pi_t = \pi_t^{trend} + \pi_t^{cycle} \tag{34}$$

$$u_t = u_t^{trend} + u_t^{cycle}. \tag{35}$$

The inflation and unemployment trends follow random walks

$$\pi_{t+1}^{trend} = \pi_t^{trend} + \sigma_{t+1}^{trend,\pi} \nu_{t+1}^{\pi} \tag{36}$$

$$u_{t+1}^{trend} = u_t^{trend} + \sigma^{trend,u} \nu_{t+1}^{u}, \tag{37}$$

where, as is standard in the literature, π_{t+1}^{trend} can be interpreted as a long-run inflation target in the central bank's policy rule and u_{t+1}^{trend} as the NAIRU. The inflation trend exhibits stochastic volatility, whereas the unemployment trend admits a constant variance.

The inflation gap π_{t+1}^{cycle} is given by a Phillips curve specification[26] following

$$\pi_{t+1}^{cycle} = \rho_{t+1}\pi_t^{cycle} + \gamma_{t+1} u_{t+1}^{cycle} + \sigma_{t+1}^{cycle,\pi} \epsilon_{t+1}^{\pi}, \tag{38}$$

where ρ_{t+1} tracks the persistence of the inflation gap and γ_{t+1} represents the contemporaneous short-run Phillips curve slope.[27] Similar to inflation trend, inflation gap variance $\sigma_{t+1}^{cycle,u}$ also exhibits stochastic volatility.

The unemployment cycle (or gap) follows

$$u_{t+1}^{cycle} = \theta_1 u_t^{cycle} + \theta_2 u_{t-1}^{cycle} + \sigma^{cycle,u} \epsilon_{t+1}^{u}, \tag{39}$$

[26] Together with the inflation trend following a random walk this makes this Phillips curve specification presented here akin to a (Hybrid) New Keynesian Phillips Curve (NKPC). See for example Stock and Watson (2008) or Cogley and Sbordone (2008).

[27] ρ_{t+1} and γ_{t+1} are truncated to have support on $(-1, 1)$. For details see Section B.3 in the appendix.

where u_{t+1}^{cycle} admits an AR(2) representation with constant parameters θ_1, θ_2 and constant variance $\sigma^{cycle,u}$.[28]

Time-varying coefficients and stochastic volatility specifications follow a random walk law of motion

$$w_{t+1} = w_t + e_{t+1}^w \quad \text{with} \quad e_{t+1}^w \sim \mathcal{N}\left(0, \sigma_w^2\right), \tag{40}$$

where $w \in \left\{\rho, \gamma, h^{trend,\pi}, h^{cycle,\pi}\right\}$ with $h_t^{trend,\pi} = \log \sigma_t^{trend,\pi}$ and $h_t^{cycle,\pi} = \log \sigma_t^{cycle,\pi}$.

A.3 Solving the Signal Extraction Problem

Forecast smoothing equation in Section 3 are motivated by noisy information expectation formation as in Woodford (2001) or Coibion and Gorodnichenko (2015). Therefore they can in principle be derived from an environment where forecasters receive noisy about the current state of the economy, and a given perceived law of motion of the economy.

Forecaster i receives noisy signals about the current state of inflation and unemployment

$$s_{i,t}^{\pi} = \pi_t + \psi_{i,t}^{\pi} \quad \text{and} \quad s_{i,t}^{u} = u_t + \psi_{i,t}^{u}, \tag{41}$$

where noise components $\psi_{i,t}^{\pi}, \psi_{i,t}^{\pi}$ are jointly normally distributed across forecasters with means zero and time-varying co-variance matrix $\Sigma_{\Psi,t}$, which is the same for all agents. Further, I assume that aggregating up over all forecasters, noise realizations cancel out.[29] Given noisy signals about inflation and unemployment,[30] it remains to define the perceived law of motion for inflation and unemployment.

In the following, I will omit sub-indices for time-variation of all parameters. Summarizing the previous subsection, the state equations (36) through (39) can be written in VAR form, by setting Matrices B and C to collect trend definitions, autocorrelation coefficients, and volatilities respectively. Σ_Ψ collects all noise volatilities.

[28]Modeling the first difference or stationary component of unemployment as an AR(2) process is common in the literature. See for example Montgomery et al. (1998).

[29]Again, this can be 'rationalized' using the law of large numbers.

[30]Among others for example Woodford (2001) assumes the noise variance to be exogenous. In different models of imperfect information this can be generalized to endogenous noise, meaning that agents choose the noise variance optimally. See for example Sims (2003).

Signal definitions in (41) can be summarized as

$$s_{i,t} = \underbrace{Hz_t}_{y_t} + \psi_{i,t}. \tag{42}$$

with $y_t = (\pi_t, u_t)'$ and matrix H[31] linking trend and cycle components to signals on inflation and unemployment and $\psi_{i,t} \sim \mathcal{N}(\mathbf{0}, \Sigma_\psi)$.

The perceived law of motion for trends and cycles, i.e. z_t, is known to all agents and is given by

$$z_{t+1} = Bz_t + C\nu_{t+1}, \tag{43}$$

with $\nu_{t+1} \sim \mathcal{N}(0, I)$.

Each forecaster i therefore computes the perceived state via Kalman mean updating

$$F_t z_t^{(i)} = \underbrace{(I - KH)}_{:=\Lambda^z} F_{t-1} z_t^{(i)} + \underbrace{KH}_{:=I-\Lambda^z} z_t + K_{t-1}\psi_{t,i}, \tag{44}$$

where K is the agent's Kalman gain and $F_t z_{t+h}^{(i)}$, is the agent's i perceived state of z_{t+h} in period t. Taking the average over all forecasters cancels out any impact of $\psi_{t,i}$ on the aggregate and delivers

$$F_t z_t = \Lambda^z F_{t-1} z_t + (I - \Lambda^z) z_t. \tag{45}$$

Notice that in the case presented here, the state-space model includes five linear states which implies that Λ^z could be potentially modeled using 25 time-varying parameters. To decrease the numbers of time-varying parameters to be estimated, but still allowing for different dynamics of expectation formation between inflation and unemployment, I allow for two time-varying parameters governing information frictions for inflation and unemployment. The next section will present the detailed setup of that matrix.

[31] Set $H = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$.

A.4 The Full Model

States and perceived states are defined by

$$z_t = \left(\pi_t^{trend}, u_t^{trend}, \pi_t^{cycle}, u_t^{cycle}, u_{t-1}^{cycle}\right)' \quad \text{and} \quad F_t z_t = \left(F_t\pi_t^{trend}, F_t u_t^{trend}, F_t\pi_t^{cycle}, F_t u_t^{cycle}, F_t u_{t-1}^{cycle}\right)' \tag{46}$$

Time-varying parameters, constant parameters, and stochastic volatilities are collected in matrices B_t and C_t.

$$B_t = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \rho_t & \gamma_t\theta_1 & \gamma_t\theta_2 \\ 0 & 0 & 0 & \theta_1 & \theta_2 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad C_t = \begin{pmatrix} \sigma_t^{trend,\pi} & 0 & 0 & 0 & 0 \\ 0 & \sigma^{trend,u} & 0 & 0 & 0 \\ 0 & 0 & \sigma_t^{cycle,\pi} & \gamma_t\sigma^{cycle,u} & 0 \\ 0 & 0 & 0 & \sigma^{cycle,u} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{47}$$

where $\sigma_t^{trend,\pi} = \exp\left(\frac{1}{2}h_t^{trend,\pi}\right)$, $\sigma_t^{cycle,\pi} = \exp\left(\frac{1}{2}h_t^{cycle,\pi}\right)$. These follow random walks given by

$$w_{t+1} = w_t + e_{t+1}^w \quad \text{with} \quad e_{t+1}^w \sim \mathcal{N}\left(0, \sigma_w^2\right), \tag{48}$$

where $w \in \left\{\rho, \gamma, h^{trend,\pi}, h^{cycle,\pi}\right\}$.

I set the matrix of information frictions to

$$\Lambda_t^z = \begin{pmatrix} \lambda_t^\pi & 0 & 0 & 0 & 0 \\ 0 & \lambda_t^u & 0 & 0 & 0 \\ 0 & 0 & \lambda_t^\pi & 0 & 0 \\ 0 & 0 & 0 & \lambda_t^u & 0 \\ 0 & 0 & 0 & 0 & \lambda_t^u \end{pmatrix}, \tag{49}$$

where I allow all two components to follow random walks with

$$\lambda_{t+1}^l = \lambda_t^l + \sigma_l e_{t+1}^l \quad \text{with} \quad l \in \{\pi; u\}. \tag{50}$$

Since back-cast data on unemployment is not available, I assume that $F_t u_{t-1}^{cycle} = \lambda_t^u F_{t-1} u_{t-1}^{cycle} + (1-\lambda_t^u)\, u_{t-1}^{cycle}$, which is enforced by setting the bottom right component of Λ_t^z equal to

λ_t^u. This implies that λ_t^u is not only governing the updating frequency of the current now-cast, but also that of the current back-cast. Therefore the parameter tracks the overall updating frequency of the information set involving unemployment.

The law of motion for states and perceived states is now given by

$$\underbrace{\begin{pmatrix} z_{t+1} \\ F_{t+1}z_{t+1} \end{pmatrix}}_{:=S_{t+1}} = \underbrace{\begin{pmatrix} B_{t+1} & 0 \\ \left(I-\Lambda_{t+1}^z\right)B_{t+1} & \Lambda_{t+1}^z B_{t+1} \end{pmatrix}}_{:=\mathcal{B}_{t+1}} \begin{pmatrix} z_t \\ F_t z_t \end{pmatrix} + \underbrace{\begin{pmatrix} C_{t+1} \\ \left(I-\Lambda_{t+1}^z\right)C_{t+1} \end{pmatrix}}_{:=\mathcal{C}_{t+1}} \nu_{t+1}, \quad (51)$$

and the observation equation by

$$\underbrace{\begin{pmatrix} y_t \\ F_t^{\text{survey}} y_{t+1} \\ \vdots \\ F_t^{\text{survey}} y_{t+5} \end{pmatrix}}_{:=\mathcal{Y}_t} = \underbrace{\begin{pmatrix} H & 0_{5x5} \\ 0_{2x5} & HB_t \\ \vdots & \vdots \\ 0_{2x5} & HB_t^5 \end{pmatrix}}_{:=\mathcal{H}_t} \begin{pmatrix} z_t \\ F_t z_t \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & & & \cdots & & & 0 \\ & 0 & & & & & \\ & & \sigma_{\zeta,\pi,1} & & & & \\ \vdots & & & \sigma_{\zeta,u,1} & & & \vdots \\ & & & & \ddots & & \\ & & & & & \sigma_{\zeta,\pi,5} & 0 \\ 0 & & & \cdots & & 0 & \sigma_{\zeta,u,5} \end{pmatrix}}_{:=\mathcal{Z}_t} \eta_t \quad (52)$$

where $y_t = (\pi_t, u_t)'$, $H = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$. Note here that current perceived states $F_t z_t$ are mapped to forecasts by using the AUM as in Section A.1 in the appendix.

A.5 Derivation of $Corr\left(F_t\pi_t^{cycle}, F_t u_t^{cycle} | \mathcal{I}_{t-1}\right)$

In this section, I derive the representation of the conditional correlation of unemployment and inflation cycle now-casts. For convenience sake, I omit all-time subscripts in time-varying parameters and PLM superscripts on parameters.

Plugging the forecasting problem's short-run Phillips curve

$$\pi_t^{cycle} = \rho \pi_{t-1}^{cycle} + \gamma u_t^{cycle} + \epsilon_t^{\pi} \tag{53}$$

into the inflation forecast smoothing equation

$$F_t \pi_t^{cycle} = \lambda^{\pi} F_{t-1} \pi_t^{cycle} + (1 - \lambda^{\pi})\, \pi_t^{cycle} \tag{54}$$

yields

$$F_t \pi_t^{cycle} = \lambda^{\pi} F_{t-1} \pi_t^{cycle} + (1 - \lambda^{\pi})\, \pi_t^{cycle} \tag{55}$$

$$\Leftrightarrow F_t \pi_t^{cycle} = \lambda^{\pi} F_{t-1} \pi_t^{cycle} + (1 - \lambda^{\pi}) \left(\rho \pi_{t-1}^{cycle} + \gamma u_t^{cycle} + \epsilon_t^{\pi} \right). \tag{56}$$

Now using the unemployment forecast smoothing equation

$$F_t u_t^{cycle} = \lambda^{u} F_{t-1} u_t^{cycle} + (1 - \lambda^{u})\, u_t^{cycle}, \tag{57}$$

and solving for u_t and plugging this into (56), we finally arrive at

$$F_t \pi_t^{cycle} = \lambda^{\pi} F_{t-1} \pi_t^{cycle} + (1 - \lambda^{\pi})\, \rho \pi_{t-1}^{cycle} + \ldots \tag{58}$$

$$- \frac{\lambda^{u} (1 - \lambda^{\pi})\, \gamma}{1 - \lambda^{u}} F_{t-1} u_t^{cycle} + \frac{(1 - \lambda^{\pi})\, \gamma}{1 - \lambda^{u}} F_t u_t^{cycle} + (1 - \lambda^{\pi})\, \epsilon_t^{\pi}. \tag{59}$$

It follows that an econometrician making use of such a regression equation to uncover the correlation of inflation and unemployment now-casts would not directly estimate γ, but

$$Corr\left(F_t \pi_t^{cycle}, F_t u_t^{cycle} | \mathcal{I}_{t-1} \right) = \frac{1 - \lambda^{\pi}}{1 - \lambda^{u}} \gamma. \tag{60}$$

B Estimation Strategy, Priors, and Calibration

B.1 Estimation Strategy

Given the state-space model in (51) and (52) collect all time-varying parameters in

$$\mathcal{V}_t = \left\{ \rho_t, \gamma_t, h_t^{trend,\pi}, h_t^{cycle,\pi}, \lambda_t^{\pi}, \lambda_t^{u} \right\}. \tag{61}$$

Linear states are summarized in

$$S_t = (z_t, F_t z_t)', \tag{62}$$

where $z_t = \left(\pi_t^{trend}, u_t^{trend}, \pi_t^{cycle}, u_t^{cycle}, u_{t-1}^{cycle}\right)'$
and $F_t z_t = \left(F_t \pi_t^{trend}, F_t u_t^{trend}, F_t \pi_t^{cycle}, F_t u_t^{cycle}, F_{t-1} u_{t-1}^{cycle}\right)'$, respectively.

Collect all constant parameters and measurement error variances in

$$\Psi = \left\{\sigma_\rho^2, \sigma_\gamma^2, \sigma_{h^{trend,\pi}}^2, \sigma_{h^{cycle,\pi}}^2, \sigma_{\lambda^\pi}^2, \sigma_{\lambda^u}^2, \theta_1, \theta_2, \sigma_{trend,u}^2, \sigma_{cycle,u}^2, \left\{\sigma_{\zeta,\pi,h}^2, \sigma_{\zeta,u,h}^2\right\}_{h=1}^5\right\}. \tag{63}$$

All parameters Ψ are then split into parameters which are either calibrated to the existing literature or computed via different means, i.e. Ψ_{calib}, and parameters which are estimated via the particle learning step, i.e. Ψ_{est}. Therefore we get

$$\Psi = \Psi_{\text{calib}} \cup \Psi_{\text{est}} \tag{64}$$

with

$$\Psi_{\text{calib}} = \left\{\sigma_\rho^2, \sigma_\gamma^2, \sigma_{\lambda^\pi}^2, \sigma_{\lambda^u}^2, \sigma_{h^{trend,\pi}}^2, \sigma_{h^{cycle,\pi}}^2, \theta_1, \theta_2, \sigma_{trend,u}^2, \sigma_{cycle,u}^2\right\}. \tag{65}$$

and

$$\Psi_{\text{est}} = \left\{\left\{\sigma_{\zeta,\pi,h}^2, \sigma_{\zeta,u,h}^2\right\}_{h=1}^5\right\}. \tag{66}$$

The state-space system in (51) and (52) can be summarized by

$$\mathcal{Y}_t = \mathcal{H}_t S_t + \mathcal{Z}_t \eta_t, \qquad \eta_t \sim \mathcal{N}(0, I) \tag{67}$$

$$S_{t+1} = \mathcal{B}_{t+1} S_t + \mathcal{C}_{t+1} \nu_{t+1}, \qquad \nu_{t+1} \sim \mathcal{N}(0, I) \tag{68}$$

$$\mathcal{V}_{t+1} \quad \text{follow Random Walks} \tag{69}$$

where $\mathcal{Y}_t$ represents real time data and survey forecasts available at time t, $\mathcal{H}_t$, $\mathcal{B}_t$, and $\mathcal{C}_t$ are matrices and contain non-linear states $\mathcal{V}_t$ and parameters Ψ_{calib}. $\mathcal{Z}_t$ contains measurement errors volatilities of survey forecasts given by Ψ_{est}.

Equations (67) through (69) define a non-linear state-space system with Gaussian innovations. Given parameters Ψ, such a system can be estimated by Sequential Monte

Carlo (SMC) methods such as Particle Filters (PF). The special structure of the problem allows for one simplification, which makes estimation tractable. Conditioning on non-linear states $\mathcal{V}_t$ the state-space system reduces to a linear Gaussian state-space model. The PF then provides filtered estimates of linear states in S_t and non-linear states in $\mathcal{V}_t$. Finally, to estimate measurement error covariances in $\mathcal{Z}_t$, I add a Particle Learning step similar to Lopes and Tsay (2011), Carvalho et al. (2010) and Mertens and Nason (2017). This adds proposal draws of measurement error variances and their corresponding sufficient statistics to the particle set in the PF. Such a procedure allows the on-line estimation of all measurement variance parameters. The remaining parameters are calibrated or estimated according to Section B.2, B.3, and B.4. In the following, I present the algorithm, a Rao-Blackwellized Auxiliary Particle Filter with Particle Learning for measurement errors (RB-APF-PL-MEAS), which is a direct extension of the RB-APF in Creal (2012) with a particle learning step for estimation of measurement error variances similar to Mertens and Nason (2017).

B.1.1 Algorithm: RB-APF-PL-MEAS

For each time t the algorithm tracks K particles $\left\{\left(S_{t-1|t-1}, \Sigma_{t-1|t-1}, \mathcal{V}_t, \Psi_{\text{est}}, s_t\right)^{(i)}\right\}_{i=1}^{K}$, where s_t are sufficient statistics of the distribution of measurement error variances. In the following set

$$z_t^{(i)} = \left(S_{t-1|t-1}, \Sigma_{t-1|t-1}, \mathcal{V}_t, \Psi_{\text{est}}, s_t\right)^{(i)}. \tag{70}$$

Over the course of the algorithm a Kalman prediction and updating step is performed. The Kalman filter equations are implemented in square root form following Carraro and Sartore (1987). This was necessary to achieve a higher degree of numerical stability, and in particular to retain the positive definiteness of $\Sigma_{t|t}$, and $\Omega_{t|t-1}$ for all i and t. To not complicate the pseudo-code any further, I will only present the standard Kalman filter equations.

Step 0

To initialize the algorithm draws proposal particles for non-linear states $\mathcal{V}_0$, sufficient statistics for the Kalman filter, i.e $S_{0|0}, C_{0|0}$, and proposal particle for measurement variances in Ψ_{est} and their corresponding sufficient statistics s_0. This results in a particle set for z_0. For details on the initialization see Section B.2.

Step 1

Using the particle set z_t run a Kalman prediction step for all particles j and compute log-likelihoods $l_t^{(i)} \propto \mathcal{P}\left(y_t|z_{t-1}^{(i)}\right)$. Herein use elements of $\mathcal{V}_{t-1}^{(i)}$ as proposals to compute time t matrices $\mathcal{B}_t^{(i)}$, $\mathcal{C}_t^{(i)}$, and $\mathcal{H}_t^{(i)}$ for all i. Use elements of $\Psi_{\text{est}}^{(i)}$ to compute $\mathcal{Z}_t^{(i)}$ for all i.

$$\Sigma_{t|t-1}^{(i)} = \mathcal{B}_t^{(i)}\Sigma_{t-1|t-1}^{(i)}\left(\mathcal{B}_t^{(i)}\right)' + \mathcal{C}_t\left(\mathcal{C}_t^{(i)}\right)', \tag{71}$$

$$\Omega_{t|t-1}^{(i)} = \mathcal{H}_t^{(i)}\Sigma_{t|t-1}^{(i)}\left(\mathcal{H}_t^{(i)}\right)' + \mathcal{Z}_t^{(i)}\left(\mathcal{Z}_t^{(i)}\right)', \tag{72}$$

$$S_{t|t-1}^{(i)} = \mathcal{B}_t^{(i)}S_{t-1|t-1}^{(i)}. \tag{73}$$

Using the predictive errors and predictive error variances compute the likelihood for each particle i via the multivariate normal density $f_N\left(y_t; \mathcal{H}_t^{(i)}S_{t|t-1}^{(i)}; \Omega_{t|t-1}^{(i)}\right)$.

Step 2

Using that compute weights by setting

$$w_t^{(i)} = \frac{f_N\left(y_t; \mathcal{H}_t^{(i)}S_{t|t-1}^{(i)}; \Omega_{t|t-1}^{(i)}\right)}{\sum_{i=1}^{K} f_N\left(y_t; \mathcal{H}_t^{(i)}S_{t|t-1}^{(i)}; \Omega_{t|t-1}^{(i)}\right)}$$

and re-sample $\tilde{z}_t$ from z_t using stratified resampling based on w_t.

Re-sampling is performed in every time step to avoid particle degeneracy. Set $z_t = \tilde{z}_t$.

Step 3

Propagate non-linear states in $\mathcal{V}_{t-1}^{(i)}$ to $\mathcal{V}_t^{(i)}$ via $p\left(\mathcal{V}_t^{(i)}|\mathcal{V}_{t-1}^{(i)}; \Psi_{\text{calib}}\right)$ with each non-linear state following a random walk.

λ_t^{π} and λ_t^{u} are drawn from a truncated normal distributed normal on the interval $(0, 1)$. Further ρ_t and γ_t are restricted to $(-0.95, 0.95)$ and $(-1, 1)$, respectively. Last, to avoid the algorithm drawing unrealistic small values for the stochastic volatilities, inflation trend and gap SV are drawn from a truncated normal distribution with an unbounded upper bound and lower bounds set to specific values. For more details see Section B.3.

Step 4

Run a full Kalman filter step for all particles i, i.e repeat the Kalman prediction step using newly drawn realizations of non-linear states and update sufficient statistics of linear states using a Kalman updating step to $S_{t|t}^{(i)}$ and $\Sigma_{t|t}^{(i)}$. Use drawn realizations of non-linear states $\mathcal{V}_t^{(i)}$ to compute matrices $\mathcal{B}_t^{(i)}$, $\mathcal{C}_t^{(i)}$, and $\mathcal{H}_t^{(i)}$ for all i.

$$\Sigma_{t|t-1}^{(i)} = \mathcal{B}_t^{(i)} \Sigma_{t-1|t-1}^{(i)} \left(\mathcal{B}_t^{(i)}\right)' + \mathcal{C}_t \left(\mathcal{C}_t^{(i)}\right)', \tag{74}$$

$$\Omega_{t|t-1}^{(i)} = \mathcal{H}_t^{(i)} \Sigma_{t|t-1}^{(i)} \left(\mathcal{H}_t^{(i)}\right)' + \mathcal{Z}_t^{(i)} \left(\mathcal{Z}_t^{(i)}\right)', \tag{75}$$

$$S_{t|t-1}^{(i)} = \mathcal{B}_t^{(i)} S_{t-1|t-1}^{(i)}, \tag{76}$$

$$\mathcal{K}_t^{(i)} = \Sigma_{t|t-1}^{(i)} \mathcal{H}_t^{(i)} \left(\Omega_{t|t-1}^{(i)}\right)^{-1}, \tag{77}$$

$$S_{t|t}^{(i)} = S_{t|t-1}^{(i)} + \mathcal{K}_t^{(i)} \left(\mathcal{Y}_t - \mathcal{H}_t^{(i)} S_{t|t-1}^{(i)}\right), \tag{78}$$

$$\Sigma_{t|t}^{(i)} = \left(I - \mathcal{K}_t^{(i)} \mathcal{H}_t^{(i)}\right) \Sigma_{t|t-1}^{(i)}. \tag{79}$$

Step 5 (Particle Learning Step)

Update sufficient statistics s_t for measurement error covariances using forecast errors $\mathcal{Y}_t - \mathcal{H}_t^{(i)} S_{t|t-1}^{(i)}$ to compute the measurement error variance's conditional posterior. Draw new realizations for Ψ_{est} via $\Psi_{\text{est}}^{(i)} \sim p\left(\Psi_{\text{est}} | s_{t+1}^{(i)}\right)$, which follows an inverse-gamma distribution. Details on the specification for the conditional posterior and on the updating of sufficient statistics can be found in B.4.

Step 6

Save linear and non-linear state estimates by computing

$$\mathcal{S}_{t|t} = \frac{1}{K} \sum_{i=1}^{K} S_{t|t}^{(i)} \quad \text{and} \quad \mathcal{V}_{t|t} = \frac{1}{K} \sum_{i=1}^{K} \mathcal{V}_t^{(i)}. \tag{80}$$

Set $t \to t+1$ and repeat Steps 1 to 6 until $t = T$.

B.2 Initialization and Priors

The number of particles K ist set to 150.000. To initialize the algorithm initial draws for non-linear states $\mathcal{V}_0$, sufficient statistics for the Kalman filter, i.e $S_{0|0}, \Sigma_{0|0}$, and proposal particle for measurement variances in Ψ_{est} and their corresponding sufficient statistics s_0 are needed. This results in a particle set for z_0. Initial K particles for γ_0, ρ_0, λ_0^π, and λ_0^u are all drawn from truncated normal distributions with

$$\gamma_0 \sim \text{trunc-}\mathcal{N}\left(0, 1; -1, 1\right) \tag{81}$$

$$\rho_0 \sim \text{trunc-}\mathcal{N}\left(0, 1; -0.95, 0.95\right) \tag{82}$$

$$\lambda_0^\pi \sim \text{trunc-}\mathcal{N}\left(0.5, 1; 0.01, 0.99\right) \tag{83}$$

$$\lambda_0^u \sim \text{trunc-}\mathcal{N}\left(0.5, 1; 0.01, 0.99\right), \tag{84}$$

where the arguments are set as (mean, variance; lower bound, upper bound). Similar, and to avoid unreasonable draws for stochastic volatilities of inflation trend and inflation gap, initial draws are also based on truncated normals:

$$h_0^{trend,\pi} \sim \text{trunc-}\mathcal{N}\left(\mu_{h,trend} - 5, \sqrt{10}; \log 0.01^2, \log 0.75^2\right) \tag{85}$$

$$h_0^{cycle,\pi} \sim \text{trunc-}\mathcal{N}\left(\mu_{h,cycle} - 5, \sqrt{10}; \log 0.05^2, \log 0.75^2\right), \tag{86}$$

with $\mu_{h,trend}$ and $\mu_{h,trend}$ being the average of their respective bound calibration, which are reflecting initial bounds of inflation trend and cycle stochastic volatilities in Stock and Watson (2007).

Given draws for non-linear states just introduced compute for all i, state-space system matrices $\mathcal{B}_0^{(i)}$, $\mathcal{C}_0^{(i)}$. To initialize the Kalman filter, extract from both matrices only the corresponding (stationary) cycle components and set those to $\tilde{\mathcal{B}}_0^{(i)}$, $\tilde{\mathcal{C}}_0^{(i)}$. Now compute the corresponding steady-state covariance matrix $\tilde{\Sigma}_{0|0}$ via solving the Lyapunov equation given by[32]

$$\tilde{\Sigma}_{0|0}^{(i)} = \tilde{\mathcal{B}}_0^{(i)} \tilde{\Sigma}_{0|0}^{(i)} \left(\tilde{\mathcal{B}}_0^{(i)}\right)' + \tilde{\mathcal{C}}_0^{(i)} \left(\tilde{\mathcal{C}}_0^{(i)}\right)', \tag{87}$$

for all particles i. Reconstruct $\Sigma_{0|0}$ by using $\tilde{\Sigma}_{0|0}$ and setting trend variances to 100^2. To draw initial proposals for the mean of linear states $S_{0|0}$ draw from a multivariate normal distribution following

$$S_{0|0}^{(i)} \sim \mathcal{N}\left(M, \Sigma_{0|0}^{(i)}\right). \tag{88}$$

[32] The code uses the matlab function 'dlyap.m'.

where

$$M = \begin{pmatrix} 2 & 5.8 & 0 & 0 & 0 & 2 & 5.8 & 0 & 0 & 0 \end{pmatrix}. \tag{89}$$

The priors for inflation and unemployment trend for data and now-casts are set to 2% for inflation, and 5.8% for unemployment, reflecting the FED St.Louis NAIRU estimate of the last quarter in 1968.

To construct $\mathcal{H}_0$ recall that

$$\Psi_{\text{est}} = \begin{pmatrix} \{\sigma_{\xi,\pi,j}\}_{j=1}^{h} & \{\sigma_{\xi,\pi,j}\}_{j=1}^{h} \end{pmatrix},$$

and draw from the corresponding prior. The prior $p\left(\Psi_{\text{est}}\right)$ is multiplicatively decomposed into independent inverse gamma distributions following

$$\begin{aligned} p\left(\sigma^2_{\xi,\pi,j}\right) &\sim i\mathcal{G}\left(\frac{\alpha_0}{2}, \frac{\beta_{0,j,\pi}}{2}\right) \quad \text{with } (\alpha_0, \beta_{0,j,\pi}) = \left(20, 18 * \bar{\sigma}^2_{\xi,\pi}\right) \text{ for } j = 1, \ldots, 5 \\ p\left(\sigma^2_{\xi,u,j}\right) &\sim i\mathcal{G}\left(\frac{\alpha_0}{2}, \frac{\beta_{0,j,u}}{2}\right) \quad \text{with } (\alpha_0, \beta_{0,j,u}) = \left(20, 18 * \bar{\sigma}^2_{\xi,u}\right) \text{ for } j = 1, \ldots, 5, \end{aligned}$$

where the prior mean is given by $\bar{\sigma}^2_{\xi,\pi} = 0.16$ and $\bar{\sigma}^2_{\xi,u} = 0.2$, respectively.

B.3 Non-linear states and Calibration

This section discusses the choice of parameters in the parameter set given by

$$\Psi_{\text{calib}} = \left\{\sigma^2_{\rho}, \sigma^2_{\gamma}, \sigma^2_{\lambda^{\pi}}, \sigma^2_{\lambda^{u}}, \sigma^2_{h^{trend,\pi}}, \sigma^2_{h^{cycle,\pi}}, \theta_1, \theta_2, \sigma^2_{trend,u}, \sigma^2_{cycle,u}\right\} \tag{90}$$

and the specification of random walk processes in

$$\mathcal{V}_t = \left\{\rho_t, \gamma_t, h_t^{trend,\pi}, h_t^{cycle,\pi}, \lambda_t^{\pi}, \lambda_t^{u}\right\}. \tag{91}$$

Non-linear states all follow random walks given by the following specifications:

$$\gamma_t \sim \text{trunc-}\mathcal{N}\left(\gamma_{t-1}, \sigma_\gamma^2; -1, 1\right) \tag{92}$$

$$\rho_t \sim \text{trunc-}\mathcal{N}\left(\rho_{t-1}, \sigma_\rho^2; -0.95, 0.95\right) \tag{93}$$

$$\lambda_t^\pi \sim \text{trunc-}\mathcal{N}\left(\lambda_{t-1}^\pi, \sigma_{\lambda^\pi}^2; 0.01, 0.99\right) \tag{94}$$

$$\lambda_t^u \sim \text{trunc-}\mathcal{N}\left(\lambda_{t-1}^u, \sigma_{\lambda^u}^2; 0.01, 0.99\right), \tag{95}$$

$$h_t^{trend,\pi} \sim \text{trunc-}\mathcal{N}\left(h_{t-1}^{trend,\pi}, \sigma_{h^{trend,\pi}}^2; \log 0.01^2, +\infty\right) \tag{96}$$

$$h_t^{cycle,\pi} \sim \text{trunc-}\mathcal{N}\left(h_{t-1}^{cycle,\pi}, \sigma_{h^{cycle,\pi}}^2; \log 0.05^2, +\infty\right). \tag{97}$$

The lower bounds for the stochastic volatility components result in a more stable algorithm. The variances of the stochastic volatility process for inflation trend and gap are common in the literature and chosen to be 0.2, following Stock and Watson (2007). Variances of remaining time-varying parameters are chosen to be 0.001 to arrive at a economically reasonable degree of time-variation.

σ_ρ^2	σ_γ^2	$\sigma_{\lambda^\pi}^2$	$\sigma_{\lambda^\pi}^2$	$\sigma_{h^{trend,\pi}}^2$	$\sigma_{h^{cycle,\pi}}^2$
0.001	0.001	0.001	0.001	0.2	0.2

Parameters for the unemployment trend and cycle specification

$$\theta_1, \theta_2, \sigma_{trend,u}^2, \sigma_{cycle,u}^2 \tag{98}$$

are estimated using real-time data on unemployment[33] and Maximum Likelihood for the following state-space model

$$u_t = u_t^{trend} + u_t^{cycle} \tag{99}$$

$$u_{t+1}^{trend} = u_t^{trend} + \sigma_{trend,u}\eta_{t+1} \qquad \eta_{t+1} \sim \mathcal{N}(0,1), \tag{100}$$

$$u_{t+1}^{cycle} = \theta_1 u_t^{cycle} + \theta_2 u_{t-1}^{cycle} + \sigma_{cycle,u}\nu_{t+1} \qquad \nu_{t+1} \sim \mathcal{N}(0,1). \tag{101}$$

[33]Civilian unemployment rate for the time period 1968Q4-2018Q3.

The likelihood is computed by the application of the Kalman filter. Optimal parameters are then given by

$$\theta_1 = 1.6955, \theta_2 = -0.7270, \sigma^2_{trend,u} = 0.1947, \sigma^2_{cycle,u} = 0.2127. \tag{102}$$

In Section C.1 to check for robustness concerning these parameter choices, I compute MDDs for a varying range of unemployment trend and cycle volatilities. The results estimated using the Kalman filter are in the range of parameters yielding the best fit in such an exercise. This implies that the parameters chosen for the unemployment process are appropriate also in an empirical model describing the average forecasting model.

B.4 Computing measurement error variances - Ψ_{est}

This section specifies the posterior updating step necessary in the the algorithm's Particle Learning step (Step 5). The goal is to arrive at an estimate of

$$\Psi_{\text{est}} = \begin{pmatrix} \{\sigma_{\xi,\pi,j}\}_{j=1}^h & \{\sigma_{\xi,\pi,j}\}_{j=1}^h \end{pmatrix},$$

which describes the variances of survey forecasts measurement errors. Given prior draws for Ψ_{est} set in Section B.2 and after resampling in (Step 2), sufficient statistics have to be updated according to the forecast error computed in the Kalman Filter (Step 4).[34] Since the prior is given by an inverse gamma distribution the posterior is also inverse gamma and the sufficient statistics can be updating following

$$\alpha_{t+1} = \alpha_t + 1, \tag{103}$$

$$\beta_{t+1,\pi,j} = \beta_{t,\pi,j} + \frac{(\mathcal{H}\mathcal{H}')_{j,j}}{\left(\Omega_{t|t-1}\right)_{j,j}} \left(\left(\mathcal{Y}_t - \mathcal{H}_t S_{t|t}\right)_{j\cdot}\right)^2, \tag{104}$$

$$\beta_{t+1,u,j} = \beta_{t,u,j} + \frac{(\mathcal{H}\mathcal{H}')_{j,j}}{\left(\Omega_{t|t-1}\right)_{j,j}} \left(\left(\mathcal{Y}_t - \mathcal{H}_t S_{t|t}\right)_{j\cdot}\right)^2, \tag{105}$$

where j corresponds to the observation equation's respective row and column which corresponds to the $j-th$ horizon inflation or unemployment survey forecast. The resulting

[34] The approach in updating measurement error sufficient statistics presented here is equivalent to Mertens and Nason (2017).

conditional posteriors to be drawn from in (Step 5) are then given by

$$p\left(\sigma^2_{\xi,\pi,j}|s_{t+1}\right) \sim i\mathcal{G}\left(\frac{\alpha_{t+1}}{2}, \frac{\beta_{t+1,\pi,j}}{2}\right)$$
$$p\left(\sigma^2_{\xi,u,j}|s_{t+1}\right) \sim i\mathcal{G}\left(\frac{\alpha_{t+1}}{2}, \frac{\beta_{t+1,u,j}}{2}\right).$$

C Robustness and additional comments

C.1 Model Selection: Unemployment Trend and Cycle Volatilities

To assure that coefficients chosen for the unemployment process are approximating the average forecaster's belief about these parameters, I compute MDDs for a range of unemployment trend and cycle volatilities.

		$\sigma_{u,\tau}$				
		0.001	0.01	0.1	0.15	0.2
$\sigma_{u,\epsilon}$	0.05	−4763.9	−4648.76	−1534.53	−977.79	−842.23
	0.1	−1960.91	−2010.27	−1343.1	−965.24	−854.23
	0.2	−1352.23	−1467.74	−990.94	−898.77	−804.36
	0.25	−1277.48	−1252.78	−996.19	−871.53	−819.74
	0.3	−1363.79	−1237.16	−936.16	−861.74	−823.66

Table 3: log MDDs for varying unemployment trend and cycle s.e.

Table 3 shows the log MDDs for unemployment trend volatilities on the horizontal and unemployment cycle innovation volatilities on the vertical axis. The largest MDD is achieved at a standard deviation 0.2 for both parameters, which is reasonably close to the values computed in Section B.2.

C.2 Exposition: Rational Inattention

This section reviews some basic ideas and terminology of rational inattention models, which motivate the parametrization of attention κ in Section 5.4. A key feature of rational inattention models is the information capacity constraint, which limits how informative a signal can be about the current state of the economy. Mathematically, this is given by the mutual information between the signal and the latent state,[35]

$$\kappa = \mathcal{I}\left(Signal, State\right). \tag{106}$$

Intuitively, mutual information tells us how informative a signal can be about some underlying state, which the agent wants to uncover. If κ is infinite, observing the signal reveals the true state. If κ is zero, the signal conveys no information about the underlying state. In models of rational inattention agents can choose the informativeness of the signal, or their degree of attention κ, subject to their preferences and an attached cost of attention.

Under linearity[36] of the model and normality of innovations, one can show that agents' attention κ_t can be tracked by their perceived signal-to-noise ratio

$$\kappa_t = \frac{1}{2}\log\left(det\left(\frac{\Sigma_{\text{Signal},t}}{\Sigma_{\text{Noise},t}}\right)\right). \tag{107}$$

So given some prior variances about the true state, the agent chooses a degree of attention, which is equivalent to choosing the noise variance attached to the signal. This maps into the noisy information model estimated in this paper, as the problem's signal-to-noise ratio equals the inverse of the model's matrix of information rigidities Λ_t^y. Therefore we can write equivalently:

$$\kappa_t = -\frac{1}{2}\log\left(det\left(\Lambda_t^y\right)\right) \tag{108}$$

$$= -\frac{1}{2}\log\left(\lambda_t^\pi\right) - \frac{1}{2}\log\left(\lambda_t^u\right) \tag{109}$$

$$= \kappa_t^\pi + \kappa_t^u. \tag{110}$$

[35]These concepts are based on insights from Information Theory. For more details see Cover and Thomas (2012)

[36]Conditionally on parameters in t the model presented in this paper is linear.